Due Wednesday, March 28th

SETS

1. List the elements in the set: \( \mathcal{P}(\mathcal{P}(\emptyset)) \). How many elements are in \( \mathcal{P}(\mathcal{P}(\emptyset)) \)? List them if you want!

2. Let \( A, B \) be subsets of some universal set. Prove that
   
   (a) \( \overline{A} = A \)
   
   (b) \( A \cap B = \overline{A} \cup \overline{B} \).
   
   (c) \( A \times B \subseteq \overline{A} \times \overline{B} \)

3. Let \( S \) and \( T \) be arbitrary sets. Prove that
   
   \[ \mathcal{P}(S) \cup \mathcal{P}(T) = \mathcal{P}(S \cup T) \text{ iff } S \subseteq T \text{ or } T \subseteq S. \]

4. Define \( n\mathbb{Z} = \{na \mid a \in \mathbb{Z} \} \) for any \( n > 0 \). Now fix some \( n, m > 0 \). Prove that \( n\mathbb{Z} \cap m\mathbb{Z} \) is a set of the form \( k\mathbb{Z} \) for some \( k > 0 \).

5. For sets \( A, B, \) and \( C \) show that
   
   \[ A \times (B \cup C) = (A \times B) \cup (A \times C). \]

6. For sets \( S \) and \( T \) define their **symmetric difference** to be the set
   
   \[ S \triangle T := S \setminus T \cup T \setminus S. \]
   
   (a) Draw the symmetric difference as a Venn Diagram.

   (b) Using part (a) conjecture and prove another (simpler) description for the symmetric difference.