Due: Monday, February 26th

1. Use the simplex method to determine the optimal solution of the following LPs:

   (a) max \( z = 2x_1 - x_2 + x_3 \) subject to

   \[
   \begin{align*}
   3x_1 + x_2 + x_3 & \leq 60 \\
   2x_1 + x_2 + 2x_3 & \leq 20 \\
   2x_1 + 2x_2 + x_3 & \leq 20 \\
   x_1, x_2, x_3 & \geq 0
   \end{align*}
   \]

   (b) max \( z = x_1 + x_2 \) subject to

   \[
   \begin{align*}
   4x_1 + x_2 & \leq 100 \\
   x_1 + x_2 & \leq 80 \\
   x_1 & \leq 40 \\
   x_1, x_2 & \geq 0
   \end{align*}
   \]

   (c) min \( z = 4x_1 - x_2 \) subject to

   \[
   \begin{align*}
   2x_1 + x_2 & \leq 8 \\
   x_2 & \leq 5 \\
   x_1 - x_2 & \leq 4 \\
   x_1, x_2 & \geq 0
   \end{align*}
   \]

2. Find an example of an LP where the “largest coefficient rule” does not result in the fewest number of pivots.

3. Assume you are told that a certain Table has exactly \( b \) basic variables and \( N \) variables in total. How many decision/slack variables must be in the corresponding LP?

4. Consider the following Final Tables for some linear programming problem. In each case, determine the set of all possible optimal solutions, i.e., what are all the possible values of the decision variables that yield the optimal value.

   (a)

   \[
   \begin{align*}
   x_5 &= 2 - 2x_2 + x_3 + x_4 \\
   x_1 &= 2 + x_2 - 2x_3 - x_4 \\
   z &= 2 - x_3 - x_4
   \end{align*}
   \]
(b)

\[
\begin{align*}
x_5 &= 10 - 2x_2 + x_4 \\
x_1 &= 20 + x_2 - 2x_3 - x_4 \\
x_6 &= 1 - 5x_3 - x_4 \\
z &= 2 - x_4.
\end{align*}
\]

5. Assume you are given an LP and you compute its final table. If all the nonbasic variables appear with < 0 coefficients in the z-line, must this LP have a unique optimal solution? Explain your answer.