

Spring 2010

Barge Problem 5

Noah vs. Lily Lily has a sequence of real numbers $\{a_1, a_2, \dots, a_n\}$. Noah and Lily are having a conversation about the sequence.

Noah: What are you doing with those numbers??

Lily: Well, for the given sequence $\{a_1, a_2, \dots, a_n\}$, I form a new sequence as follows: First, I pick a real number b , then I subtract b from each of the a_1, a_2, \dots, a_n then I take the absolute values of all these numbers.

Noah: So your new sequence is just

$$|a_1 - b|, |a_2 - b|, \dots, |a_n - b|?$$

Lily: Yes. Now I can repeat this process with the new sequence. I can do this over and over and over!

Noah: How exciting for you. Each time, you can pick any b that you like, but you have to subtract that b from every number in the current sequence and then take absolute values. What are you trying to do?

Lily: I'm trying to get all of the numbers equal to 0.

Noah: That's not always possible!

Lily: Yes it is!

Noah: I quit. I want a hamster!

Your job is to figure out who's right. If it's Lily, show that, for any given sequence $\{a_1, a_2, \dots, a_n\}$, it is always possible to get all the numbers (eventually) to be 0. If it's Noah, give a specific counterexample, along with a proof that it's not possible for your sequence. As usual, write enough so that someone can grade this and give you lots of credit.

Deadline: Saturday, April 3 at 6:00 a.m.