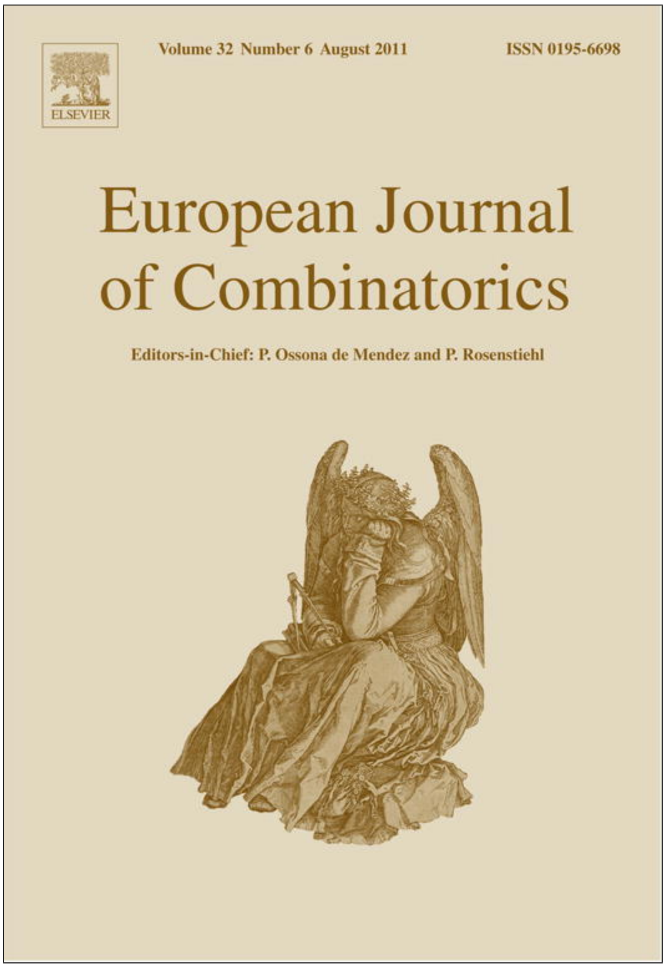
Volume 32 Number 6 August 2011
ISSN 0195-6698
ELSEVIER
European Journal of Combinatorics
Editors-in-Chief: P. Ossona de Mendez and P. Rosenstiehl

European Journal of Combinatorics 32 (2011) 712–721

Contents lists available at ScienceDirect

European Journal of Combinatorics

journal homepage: www.elsevier.com/locate/ejc

Thomas H. Brylawski (1944–2007)

Gary Gordon [a], Jennifer McNulty [b]

[a] Lafayette College, Mathematics Department, Easton, PA 18042, United States
[b] Department of Mathematical Sciences, The University of Montana, Missoula, MT 59812, United States

ARTICLE INFO

Article history:
Available online 5 March 2011

ABSTRACT

This article is a tribute to Thomas H. Brylawski's mathematics and his life.

Dedicato alla memoria di Tom Brylawski, con grande affetto e gratitudine. La sua impronta sul mondo è stata profonda.

Thomas H. Brylawski was one of the most engaging mathematicians of his generation, full of life and wit. He made an enormous impact on all who were lucky enough to know him. He had tremendous

E-mail address: gordong@lafayette.edu (G. Gordon).

0195-6698/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ejc.2011.02.009

energy for anything that caught his attention (once staying awake for 36 h to construct an algorithm to solve Rubik's cube), and that energy went into his talks, his conversations, his children, and his mathematics. Many of his papers in matroid theory from the 1970s and 80s are still widely cited. Moreover, Tom had an uncanny ability to draw in an audience with his humor and unique speaking style, as anyone who witnessed one of his talks can attest.

1. Early years

Tom grew up in Washington, D.C., and his friends remember his precocious talent in mathematics, his very quick wit, and his complete lack of organization. This last part of his personality was obvious to his friends early on; in 6th grade, Tom's teacher had to ask one of Tom's friends to straighten Tom's desk. This was a lifelong affliction — many years later, those visiting his office in Chapel Hill would be unable to locate his desk.

His friends recall how Tom managed to find the humor in just about any situation. He loved games and puzzles of all sorts. One summer, while in high school, he worked for an operations research company, where he was supposed to be modeling elevator waiting time in multi-story buildings as a Markov process. A friend who worked with him notes that the real achievements that summer were, however, in the card game, hearts.

Tom was an undergraduate at MIT, majoring in math. Early in his undergraduate career, Tom took a circuits class with C.L. Liu. He loved the material, which included lots of combinatorial problems (for instance, finding the resistance in an $n \times n$ grid). On the other hand, he also signed up for a calculus class with Gian-Carlo Rota, who would later become his Ph.D. advisor. Rota walked in and declared that there were simply too many students in the class, so he began lecturing on algebraic topology. This had the desired effect — Tom (and several others) dropped the class.

While at MIT, he enjoyed a wide variety of non-mathematical activities, including his fondness for creating mathematically precise large oil paintings (according to one of his fraternity brothers). His interest in art was serious, and it would take a mathematical direction later in his career. Tom had many hobbies, and he spent hours playing cards, usually bridge. He was also a good athlete, playing tennis and an intimidating brand of ping pong. He also loved rock and roll, especially Elvis. Tom played guitar and sang enthusiastically his entire life.

Tom went to Dartmouth for his Ph.D. While still in graduate school, he married and started a family. Working with Rota at MIT involved commuting from Hanover to Cambridge, which he did frequently. During this time he met several combinatorialists who would become good friends, including Henry Crapo, Curtis Greene, Richard Stanley, Neil White, and Tom Zaslavsky. Since he was a student at Dartmouth, he still needed an advisor on the faculty of Dartmouth; Bob Norman was Tom's official advisor.

2. Mathematical contributions

Tom was a major contributor to matroid theory in the 1970s and 80s, and much of that work is an important foundation for current research. He wrote fundamental papers on several important topics, including polynomial invariants, matroid constructions, lattices associated with matroids, matroid "cryptomorphisms" (a term coined by Birkhoff to describe structures that were *secretly* isomorphic), matroid reconstruction, and matroid representations. Rota was a strong advocate for the geometric approach to matroids, and his influence on Tom was profound. Tom imparted this perspective to his students and young researchers he worked with. James Oxley worked with Tom as a post-doctoral student in the late 1970s. In the preface of his text on matroids [12], Oxley writes, "This book attempts to blend Welsh's very graph-theoretic approach to matroids with the geometric approach of Rota's school I learnt from Brylawski".

2.1. Matroid publications

Tom's early papers are long, detailed and comprehensive. His Ph.D. dissertation, "The Tutte–Grothendieck Ring", laid much of the foundation for the algebraic underpinnings of the Tutte polynomial. Tom's early work used tools from category theory, and he believed abstraction is useful when it is motivated by combinatorial problems of wider interest. For instance, in the detailed and extremely well-motivated introduction of (3), he writes "The present work is born in the application to combinatorial theory of some techniques that have been successfully used in the field of commutative algebra and algebraic topology. ... By taking an abstract point of view, we carry over to combinatorial pre-geometries [matroids] constructions which resemble the Grothendieck group and ring". This approach allowed Tom to study evaluations and coefficients of the Tutte polynomial (a relatively unstudied invariant in 1972).

Tom's three major papers (1, 3, 10), all of which appear in *Transactions of the American Mathematical Society*, build a coherent theory of matroids by unifying important results of Tutte [17], Crapo [3], Stanley [16], and others. All of these papers are still widely referenced; *Web of Science* lists 173 separate citations of these three papers, at least 35 of which have appeared in the last five years. The first two of these papers focus on the interrelations among matroid invariants, especially the Tutte polynomial. The Tutte polynomial results are fundamental, and current work on this invariant builds from Tom's foundation. Tom was certainly one of the first people to realize how important this invariant was. This work is quite algebraic in nature; for instance, he found a basis for all linear relations satisfied by the coefficients of the Tutte polynomial.

Studying the connection between matroid constructions and factoring properties of the characteristic polynomial is the theme of (10). This paper examines modular flats from the lattice-theoretic perspective, and succeeds in explaining why these flats are so important. In his review of (10) in *Mathematical Reviews*, John Mason writes, "This is a substantial paper on the properties induced on a geometry by the presence of a modular flat. It is extremely detailed and comprehensive". It also leads to a more extensive study of matroid constructions, including pushouts, tensor products, generalized parallel connections, and other ways to create new matroids. Tom's expository work on this subject (34) is the standard reference on the topic.

Tom's papers (22, 24, 27) on intersection theory are an attempt to unify the classic geometric problems studied in finite projective geometry with the study of combinatorial invariants. For example, connections between the codeweight polynomial of a linear code and the Tutte polynomial of the associated linear matroid were discovered by Dowling [5] and extended by Greene [6]. Intersection theory places the results from these papers in a general setting. In the introduction to (22), Tom is explicit about this motivation: "Classical nineteenth century work in finite geometry has branched in this century into two directions: on the one hand there is the analytic (and algebraic) exploration of finite projective spaces pioneered by such mathematicians as Veblen, Bose, B. Segre, and others, while on the other hand there is the theory of matroids, a synthetic, combinatorially abstract approach begun by Whitney, Tutte and others. ... It is the intention of this paper to merge these two areas by giving a matroid-theoretic foundation to combinatorial intersection theory".

Tom introduced the *polychromate* in (27), a multivariate generalization of the Tutte polynomial. He used this to create examples of graphs with arbitrarily high connectivity having the same (ordinary) Tutte polynomial. This work anticipated current work on graphs and matroids determined by their Tutte polynomials (see, for example, [4] for a survey). In fact, the polychromate has been applied to rooted trees [2] and shown to be equivalent to two other multivariate polynomials in [11]. A recent treatment of multivariate Tutte polynomials was completed by Sokal [15].

Tom's interest in invariants and structural properties of geometric lattices led him to study the broken-circuit complex of a matroid in (18, 28) (the latter paper is joint with James Oxley). Wilf introduced this complex in [18], applying it to the chromatic polynomial of a graph. Tom generalized this to the characteristic polynomial of a matroid. He used tools of combinatorial topology to prove the complex is a cone, and he gave three different proofs that the coefficients of the characteristic polynomial are encoded by the reduced broken-circuit complex. In fact, Tom often includes two or three different proofs of the same result in his papers, a choice that helps his readers understand a topic more completely. This carried over into his teaching.

Connections between polynomials and lattices are also treated in (17, 25). His research in lattices extended to non-geometric lattices, that is, lattices that do not arise from matroids. In (5), he studied the very important lattice of partitions of an integer, computing its Möbius function and proving several theorems about its structure.

Matroid representations are the theme for the papers (11, 16, 30, 33). He published (16) with his first Ph.D. student, Dean Lucas; the theorem that matroid representations are unique over the field $GF(3)$ has been a key tool in proving excluded-minor theorems. The technique used in creating matroids with finite characteristic sets appearing in (25) was motivated by the unpublished work of Ralph Reid, who was working in biochemistry at UNC in the late 1970s. Tom taught Reid about matroids when Reid was an undergraduate at UNC, and Reid proved the excluded-minor characterization of ternary matroids. (Reid never published his proof, but presented it at a combinatorics conference in 1971 at Bowdoin College. Bixby gives Reid credit for this theorem in [1].) This work also influenced Kahn, who proved that any finite set of primes is the characteristic set of some matroid [8].

Tom's research papers, on a variety of matroid topics, show how much Tom was influenced by the people and mathematics around him. For example, in (14), Tom mentions the motivation for the paper came from a "remark made at a CUPM Geometry Conference... that matroids do not arise naturally" in the context of convex sets. In this paper, Tom showed the Radon number is a matroid invariant, thus showing that matroids and convex sets are intimately related.

Tom kept up on the latest research in matroid theory, and, more generally, in mathematics. In 1993, Tom attended a conference on oriented matroids. Despite being new to the field, Tom co-authored a paper (41) with Günter Ziegler on the representation of dual pairs of oriented matroids, a problem that required combining two ways of viewing an oriented matroid. Tom's last set of papers (44, 45) arose from his interactions with his colleague Alexander Varchenko. Although Varchenko's research is far from matroid theory, the two were able to find common ground by constructing a matroid analog of the formula for the determinant of the Shapovalov form in representation theory. It is especially fitting that Tom's final publication appeared in a volume dedicated to his thesis advisor, G.-C. Rota.

Tom's papers were written to be read, although many of them are technical. He included examples throughout his papers, and he often included a final section on research problems. Many of these problems inspired his students and his colleagues. For instance, Tom conjectured that the Tutte polynomial of a connected matroid M is irreducible in the ring $\mathbb{Z}[x, y]$. This conjecture was proved by Merino, de Meir and Noy in 2001 [10].

2.2. Expository and other writing

In an entertaining essay, G.-C. Rota observes that mathematicians are more likely to be remembered for their expository work than their published research [13]. Tom understood this; he published several influential expository papers as chapters in books, and he also published a monograph on matroids (23) (jointly with Doug Kelly). His chapter on matroid constructions (34) was a key contribution to the Theory of Matroids series of books published by Cambridge in the 1980s. Neil White, Tom's friend and colleague, was the editor of the series, and he remarks on Tom's work, "When the book (later series of books) on Theory of Matroids which I was editing for Cambridge University Press was in serious danger of imploding, Tom produced in a very short period of time both the central chapter on Constructions (34) and the appendix on Matroid Cryptomorphisms (35). These are certainly the most often consulted and referenced contributions to the whole series, and Tom's effort at that time was the turning point of the whole project".

Tom's contribution to that series also included an extremely thorough chapter on the Tutte polynomial (40) (joint with James Oxley). This chapter is an excellent resource for researchers, and it has been cited over 100 times in the literature. In fact, Tutte himself reviewed this chapter for *Mathematical Reviews*, writing "The reviewer, having once worked on that polynomial himself, is awed by this exposition of its present importance in combinatorial theory". The chapter (40) is based (in part) on Tom's earlier work (31), a 150-page paper that contains much new material. One important result of (31) is a formula for the Tutte polynomial of the tensor product of two matroids. This result played a key role in the seminal paper of Jaeger et al. [7] on the computational complexity of evaluations of the Tutte polynomial.

Tom loved problems, and he included an exceptionally diverse and well-researched collection of problems at the end of both (34) and (40), as well as throughout his book (23). These range from computational to research level, and they often make a point (leading the solver to construct a counterexample, or showing how a natural question can be resolved, for example). The problems and very short exposition made (23) an invaluable resource for many students who learned matroid theory in the 1980s. The text also includes previously unpublished research of Ralph Reid on characteristic sets of matroids.

Tom's expository writing often contained original work not previously published. This is the case in (33), where Tom works through a careful coordinatization of the Dilworth truncation of a representable matroid (this was the first published proof of this theorem, due to Mason [9], and it explains why any realizable matroid characteristic set can be realized by a rank-3 matroid). This is also true of his appendix (35) on cryptomorphisms. This is organized in an appealing, chart-like fashion (Tom loved charts, and cryptomorphisms), which is one reason it remains a useful tool for researchers.

3. Teaching, mentoring, and talks

Tom was an inspiring and demanding teacher, with high expectations and tough standards. His lectures were extensions of his personality — very fast, very loud, sometimes very funny, and frequently hard to follow, but ultimately rewarding. Robert Bryant, Director of MSRI, was a graduate student at UNC; he writes "Tom was a wonderful teacher for me in my first year in graduate school". Tom had a great impact on his students. While a senior in high school, Liz McMahon (UNC Ph.D.) took Tom's Calculus class — the first class Tom taught at UNC. Liz was influenced greatly by Tom; she says "I think that part of why I'm a mathematician today is his influence". Tom's last Ph.D. student Navin Vembar agrees, "His influence on me is hard to overstate. It was he who convinced me to come back and complete my Ph.D., which has opened up opportunities that simply would not have existed before".

As a confirmed night owl, his 8 a.m. teaching assignment his first semester was a challenge. The department head backed off the following semester, giving Tom a 9 a.m. teaching assignment. Tom's response was typical: "Now I need to stay up till 9!" Tom was able to find the ideal sleeping situation during one visit to Italy. On that extended visit, he habitually slept from 7 to 11. When asked if this was a morning or evening schedule, he replied 'both'. He would wake at 11 p.m., work all night, sleep from 7 to 11 in the morning, then socialize and talk mathematics with his colleagues in the afternoon, going to sleep at 7 in the evening.

Tom enjoyed teaching graduate students, and for may years, he taught the graduate combinatorics class and a graduate class in matroids. All of these courses were packed with interesting problems. Although he taught the graduate combinatorics class many times, he used a different text each time. It was not unusual to have several faculty or post-doctoral students sitting in on a class, and he occasionally team taught classes. He covered material rapidly, and he expected students to do whatever they needed to in order to keep up. He once offered an A in a graduate combinatorics class to anyone who could devise a combinatorial proof to an identity. One student succeeded, to Tom's surprise, and he graciously kept his word (although the semester was near its end when Tom made the offer).

As a Ph.D. advisor, Tom was extremely generous with his time and his ideas. It was not unusual to start a session with Tom in the early afternoon (they never started in the morning) that extended to dinner and beyond. 'Beyond' usually included other guests, beer, lots of interesting conversation and – always – tremendous energy and laughter along with wonderful problems to think about.

Tom was also a mentor for young researchers. Joe Bonin invited Tom to give a talk at George Washington University. The talk started at 1:00, and Tom started by saying, "I assume you run colloquia here as we do at Chapel Hill — ours end at 5 o'clock". That elicited nervous laughter, and Tom ended at the scheduled time of 2:00, but he continued to talk with the GW combinatorics group for another 3 h. Joe recalls, "much of the work I did on Tutte polynomials in the years after that was significantly influenced by that afternoon with Tom".

Tom also served the profession as an editor, a referee and a reviewer. He was an editor for *Proceedings of the American Mathematics Society* and *Discrete Mathematics*. He took this job quite seriously, and that dedication extended to the reviews he published in *Mathematical Reviews*. Recognizing the profound significance of Paul Seymour's regular-matroids-decomposition paper [14], Tom wrote a detailed review of it explaining both the main results and the methods used in their proofs. On the other hand, he could certainly take an author to task for sloppy work. Of one paper, he wrote:

> Six propositions follow of which only two are correct .... The remaining propositions with their counterexamples in parentheses are included in this review not because they are deemed important but to discourage future papers with a litany of undemonstrated, ill-thought-out results.

Tom could be very funny, and his talks were events — he spoke loud, fast and he was constantly in motion. "He had a reputation for talking faster than most people thought, and listening to his lectures was often akin to riding a tornado", recalls Scott Provan, one of Tom's colleagues. Tom gave a memorable talk on July 4, 1995, at a matroid workshop in Seattle. This talk was a tour de force; very funny, highly enjoyable, with a new and interesting formula for matroids as the mathematical content. The talk made quite an impression: Geoff Whittle remembers "It was the most entertaining mathematics talk I have ever attended. Tom opened the talk with Whitney's declarations of independence!"

Tom's final public presentation was 'A partially anecdotal history of matroid theory', delivered at the Matroids in Montana workshop in November of 2006. His talk was, as usual, funny and very informative, and it was the highlight of that meeting. A videotaped projection of the talk was the keynote of the Tom Brylawski Memorial Conference held in Chapel Hill in October of 2008, where it was viewed by a room full of Tom's friends and colleagues.

One of Tom's greatest strengths as a mathematician was his tremendous insight into a wide range of problems and his ability to pose interesting questions in a variety of areas. When attending a lecture, Tom always seemed to have something substantial to contribute, a question or an idea for further research. "Tom's steady attendance at our departmental colloquia was exceeded by none....time and again he would amaze me in the way he was able to follow the general drift of a talk and ask an astute question at the end", recalls colleague Bob Proctor.

3.1. *Math and art*

Tom had a serious interest in art, and some of this interest manifested itself in the mathematics underlying the 17 wallpaper groups. Indeed, he lectured on this topic at the National Gallery of Art. He taught "Math & Art" courses designed for students at a variety of levels, for those with no mathematical background, for education majors, for art majors, for Elder Hostel groups, and for graduate students (and interested colleagues) in a research seminar setting. The long preprint (42) (joint with Jeff Sheats) is a detailed analysis of symmetry breakers via a parameter space. He also published pedagogical work in the two *experiments* (8, 9).

He often commented that the retirees he taught in his Elder Hostel courses were especially motivated by the topic, and the women who had quilted were very good at identifying the symmetry groups. He had a real talent for visualizing abstract group-theoretic properties, and much of his original work in this area was in this spirit. He claimed he could get undergraduates with no mathematical background to understand semidirect products of groups "visually". This material formed the basis of a research-level MAA minicourse he taught, and his copious notes are a valuable resource for anyone interested in this topic.

Tom's interest in symmetry led to some interesting mathematical artwork in his home. The first pair of 10×10 orthogonal Latin squares were discovered by Bose, Shrikhande, and Parker in Chapel Hill in 1959, and Tom proudly displayed his tribute to their discovery (see Fig. 2). He also painted a 'squared square' (Fig. 1) and a counterexample to Kempe's flawed 'proof' of the four color theorem. His 'group coffee table' (see Fig. 3) illustrates how 7 of the crystallographic groups can be generated by regions bounded by mirrors. Visitors to Tom's house were required to stick their heads into each of the 7 regions.

In the early 1980's, Tom advised Charles Gunn's Master's thesis "A Computer Implementation of the 17 Euclidean Wallpaper Groups". Charles writes, "I believe that Tom, knowing my passion for computer graphics, and being aware of the difficulty I had in uniting that with my other passion for

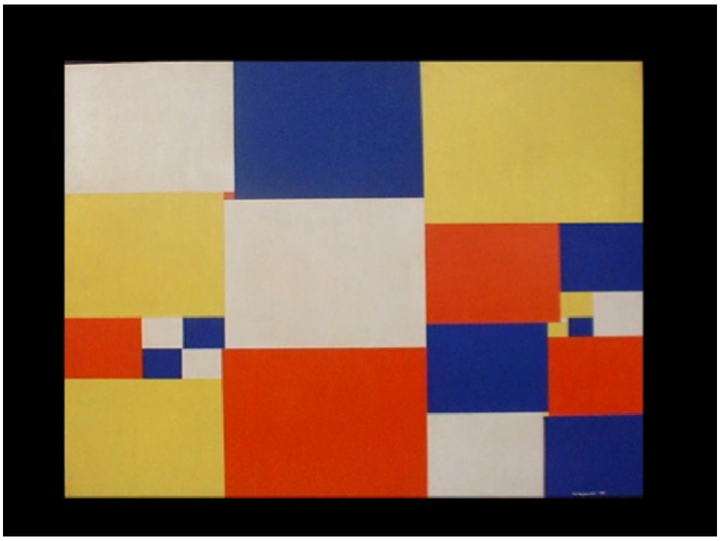

Fig. 1. Squared square, painted by Tom Brylawski.

Fig. 2. 10×10 Orthogonal Latin Squares, painted by Tom Brylawski.

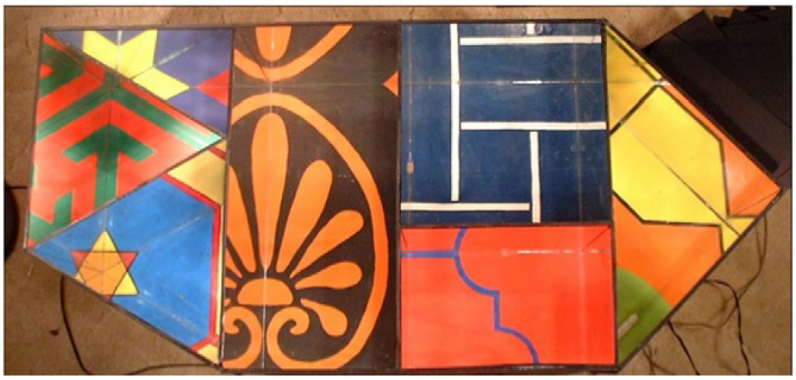

Fig. 3. Tom's innovative 'group coffee table' can show 7 of the 17 crystallographic groups.

mathematics, suggested the project himself. He was a helpful and interested partner in fulfilling this project''.

Tom also found connections between symmetry groups and matroids. In his paper (12), he constructed a matroid based on one of the Archimedean solids, the *rhombicuboctahedron*. He proved the associated matroid M is *homogeneous*, i.e., $M/p \cong M/p'$ for all points p, p', but he also showed M

does not have a transitive automorphism group. He used this matroid to produce a counterexample to a reconstruction conjecture: It is not possible to reconstruct a matroid from the deck of one-point contractions.

4. Other interests

Tom's interests outside mathematics were very wide, ranging from high culture to pop culture. He was a serious art collector; he also collected movie posters (in a variety of languages, but mostly Italian); he was devoted to the old television show The Andy Griffith Show (he was once interviewed by Dan Rather at an Andy Griffith convention); he was a serious Contract Bridge player; he was a die-hard Washington Redskins fan; and he loved his North Carolina Tar Heel basketball teams. Tom led several faculty College Bowl teams in the 1970s, and he also excelled at word games and various cryptic and crossword puzzles. But his lifelong love of rock music in general and Elvis Presley in particular stands out. Many UNC students and faculty remember his Elvis impersonations, complete with guitar, usually performed late in the evening at the department Christmas party.

Tom loved Italy and all things Italian (especially his wife Bruna). He spent many happy summers there, and organized a Matroid Theory Conference at Lake Como in the summer of 1982. Tom worked hard on his Italian, both as a writer and a speaker, and he even gave some lectures in Italian. "In one of these early trips, his knowledge of the Italian language was still minimal, and he wanted to improve quickly. Therefore he decided to give his talk titled "The Mathematics of Watergate" in Italian. His preparation consisted of going through the English text of the talk virtually word for word, consulting his Italian dictionary and grammar text frequently to produce an Italian translation for oral delivery". recalls Rhodes Peele one of Tom's Ph.D. students.

Tom played a unique role in the mathematics department in Chapel Hill. Tom often carpooled with his friend and colleague Sue Goodman. She remembers him fondly; "In his inimitable way, Tom was the heart of the department. He prodded us to do the many things we should have been doing anyway but didn't always: pledging support to charities, joining a speaker for a drink or dinner, staying in touch with old friends. He attended every colloquium, every coffee hour, every social gathering. He never forgot anything anyone said to him, even when we wished he would. We miss him in ways we never imagined, for things we didn't always appreciate at the time, but should have". Tom also enjoyed meeting new people, including faculty, visitors and graduate students. Dan Curtin (UNC PhD) also remembers Tom as "far and away the most welcoming of new people".

Tom was devoted to his two sons, David and Michael. He was very proud of their accomplishments, and it took very little prodding to get him to talk about them. After they left home, he would frequently call, usually around 3 a.m., when Tom was always wide awake. But he had an exceptional ability to relate to any child, and many a math gathering included a group of children surrounding or hanging from Tom.

What made Tom so endearing and unique was his honesty, his love of life, his passion, his humor, and, above all, his humanity. We remember Tom's lasting contributions to mathematics, but we also treasure his friendship. He inspired his students, his colleagues, and his friends, of whom there were many. The authors are profoundly thankful to have had Tom play such a meaningful role in their lives.

Publications of Thomas H. Brylawski

(1) "A combinatorial model for series–parallel networks", *Trans. Amer. Math. Soc.* **154** (1971) 1–22.
(2) "The Möbius function on geometric lattices as a decomposition invariant", *Möbius algebras* (Proc. Conf., Univ. Waterloo, Waterloo, Ont., 1971), pp. 143–148. Univ. Waterloo, Waterloo, Ont., 1971.
(3) "A decomposition for combinatorial geometries", *Trans. Amer. Math. Soc.* **171** (1972), 235–282.
(4) "The Tutte–Grothendieck ring", *Algebra Universalis* **2** (1972), 375–388.
(5) "The lattice of integer partitions", *Discrete Math.* **6** (1973), 201–219.
(6) "Some properties of basic families of subsets", *Discrete Math.* **6** (1973), 333–341.
(7) "Reconstructing combinatorial geometries", *Graphs and combinatorics* (Proc. Capital Conf., George Washington Univ., Washington, D.C., 1973), pp. 226–235. Lecture Notes in Math., Vol. 406, Springer, Berlin, 1974.
(8) "Experiment 34: An Appendix in color mixing", *Experiments in Physics and Art* (1974), 141–142.

(9) "Experiment 36: Mirrors and symmetry", *Experiments in Physics and Art* (1974), 156–192.
(10) "Modular constructions for combinatorial geometries", *Trans. Amer. Math. Soc.* **203** (1975), 1–44.
(11) "A note on Tutte's unimodular representation theorem", *Proc. Amer. Math. Soc.* **52** (1975), 499–502.
(12) "On the nonreconstructibility of combinatorial geometries", *J. Combinatorial Theory Ser. B* **19** (1975) 72–76.
(13) "An affine representation for transversal geometries", *Studies in Appl. Math.* **54** (1975) 143–160.
(14) "A combinatorial perspective on the Radon convexity theorem", *Geometriae Dedicata* **5** (1976) 459–466.
(15) "A determinantal identity for resistive networks", *SIAM J. Appl. Math.* **32** (1977) 443–449.
(16) (with T. D. Lucas) "Uniquely representable combinatorial geometries", *Atti dei Convegni Lincei* **17** Tomo I 1976 83–104.
(17) "Connected matroids with the smallest Whitney numbers", *Discrete Math.* **18** (1977), no. 3, 243–252.
(18) "The broken-circuit complex", *Trans. Amer. Math. Soc.* **234** (1977) 417–433.
(19) "Combinatorial theory" (article and original artwork), *Encyclopedia of Science and Technology* 4th and 5th editions, 8 pages, McGraw-Hill, 1977 and 1980.
(20) *University of Rome Lecture Series* 1977.
  (a) "Geometria combinatorie e loro applicazioni",
  (b) "Funzioni di Möbius",
  (c) "Teoria dei codici e matroidi",
  (d) "Matroidi coordinabili".
(21) (with D. Kelly) "Matroids and combinatorial geometries", *Studies in combinatorics*, pp. 179–217. MAA Studies in Math., **17** Math. Assoc. America, Washington, D.C., 1978.
(22) "Intersection theory for embeddings of matroids into uniform geometries", *Stud. Appl. Math.* **61** (1979), no. 3, 211–244.
(23) (with D. Kelly) "Matroids and combinatorial geometries", *Carolina Lecture Series*. University of North Carolina, Department of Mathematics, Chapel Hill, N.C., 1980. iv+149 pp.
(24) "The affine dimension of the space of intersection matrices", *Rend. Mat.* (6) **13** (1980), 59–68.
(25) (with J. Oxley) "Several identities for the characteristic polynomial of a combinatorial geometry", *Discrete Math.* **31** (1980), 161–170.
(26) (with P. M. Lo Re, F. Mazzocca, and D. Olanda) "Some applications of the intersection theory to Galois geometry", (Italian) *Ricerche Mat.* **29** (1980), no. 1, 65–84.
(27) "Intersection theory for graphs", *J. Combin. Theory Ser. B* **30** (1981), no. 2, 233–246.
(28) (with J. Oxley) "The broken-circuit complex: its structure and factorizations", *European J. Combin.* **2** (1981), no. 2, 107–121.
(29) "Hyperplane reconstruction of the Tutte polynomial of a geometric lattice", *Discrete Math.* **35** (1981), 25–38.
(30) "Finite prime-field characteristic sets for planar configurations", *Linear Algebra Appl.* **46** (1982), 155–176.
(31) "The Tutte polynomial. I. General theory", *Matroid theory and its applications,* 125–275, Liguori, Naples, 1982.
(32) "Iterated parallel union of matroids", *Rend. Sem. Mat. Fis. Milano* **53** (1983), 229–244 (1986).
(33) "Coordinatizing the Dilworth truncation", Matroid theory (Szeged, 1982), 61–95, *Colloq. Math. Soc. Jnos Bolyai,* **40** North-Holland, Amsterdam, 1985.
(34) "Constructions", Theory of matroids, 127–223, *Encyclopedia Math. Appl.,* **26** Cambridge Univ. Press, Cambridge, 1986.
(35) "Appendix of matroid cryptomorphisms, Theory of matroids, 298–316, *Encyclopedia Math. Appl.*, **26** Cambridge Univ. Press, Cambridge, 1986.
(36) "Blocking sets and the Möbius function", *Symposia Mathematica,* Vol. XXVIII (Rome, 1983), 231–249, Sympos. Math., XXVIII, Academic Press, London, 1986.
(37) (with E. Dieter) 'Exchange systems", *Discrete Math.* **69** (1988), no. 2, 123–151.
(38) "Greedy families for linear objective functions", *Stud. Appl. Math.* **84** (1991), no. 3, 221–229.

(39) "Matroid blocking sets", Combinatorics '88, Vol. 2 (Ravello, 1988), 11–37, it Res. Lecture Notes Math., Mediterranean, Rende, 1991.
(40) (with J. Oxley) "The Tutte polynomial and its applications", Matroid applications, 123–225, *Encyclopedia Math. Appl.,* **40** Cambridge Univ. Press, Cambridge, 1992.
(41) (with G. Ziegler) "Topological representation of dual pairs of oriented matroids", *Discrete Comput. Geom.* **10** (1993), no. 3, 237–240.
(42) (with J. Sheats) "Stratified parameter spaces for isohedral dirichlet tilings", Preprint 1994.
(43) "A conjecture on rank-preserving weak map images", (in Open Problems). *Contemporary Mathematics* **197** 1996 p. 412.
(44) (with A. Varchenko) "The determinant formula for a matroid bilinear form", *Adv. Math.* **129** (1997), no. 1, 1–24.
(45) "A Möbius identity arising from modularity in a matroid bilinear form", In memory of Gian-Carlo Rota., *J. Combin. Theory Ser. A* **91** (2000), 622–639.

Student	Year	Dissertation title
T. Dean Lucas	1974	Mappings in combinatorial geometry
Ulrich Faigle	1977	Geometries on partially ordered sets
Rhodes Peele	1978	Whitman number upperbounds for some geometric lattices
Gary Gordon	1983	Representations of matroids over prime fields
Jennifer McNulty	1993	Affine hyperplane arrangements and oriented matroids
Navin Vembar	2003	Oriented matroid integer chains

Ph.D. students of Thomas H. Brylawski.

Acknowledgements

The authors would like to thank James Oxley for his very helpful comments, Lorenzo Traldi and Alessandro Giovannelli for help with the dedication, and the many mathematicians and friends of Tom who provided interesting comments and anecdotes about his life. Some more of these are collected on a memorial website: http://www.math.unc.edu/Faculty/thb/Remembs.html.

References

[1] R. Bixby, On Reid's characterization of the ternary matroids, J. Combin. Theory Ser. B 26 (1979) 174–204.
[2] B. Bollobás, O. Riordan, Polychromatic polynomials, Discrete Math, 219 1–7.
[3] H. Crapo, A higher invariant for matroids, J. Combin. Theory 2 (1967) 406–417.
[4] A. de Mier, M. Noy, On matroids determined by their Tutte polynomials, Discrete Math. 302 (2005) 52–76.
[5] T. Dowling, Codes, packings and the critical problem, Atti del Convegno di Geometria Combinatoria e sue Applicazioni (Univ. Perugia, Perugia, 1970), Ist. Mat., Univ. Perugia, Perugia, 1971, pp. 209–224.
[6] C. Greene, Weight enumeration and the geometry of linear codes, Studies Appl. Math. 55 (1976) 119–128.
[7] F. Jaeger, D. Vertigan, D. Welsh, On the computational complexity of the Jones and Tutte polynomials, Math. Proc. Cambridge Philos. Soc. 108 (1990) 35–53.
[8] J. Kahn, Characteristic sets of matroids, J. Lon. Math. Soc. 26 (1982) 207–217.
[9] J. Mason, Glueing matroids together: a study of Dilworth truncations and matroid analogues of exterior and symmetric powers, in: Algebraic Methods in Graph Theory, Vol. I, II (Szeged, 1978), in: Colloq. Math. Soc. Jnos Bolyai, vol. 25, North-Holland, Amsterdam-New York, 1981, pp. 519–561.
[10] C. Merino, A. de Mier, M. Noy, Irreducibility of the Tutte polynomial of a connected matroid, J. Combin. Theory Ser. B 83 (2001) 298–304.
[11] C. Merino, S. Noble, The equivalence of two graph polynomials and a symmetric function, Combin. Probab. Comput. 18 (2009) 601–615.
[12] J. Oxley, Matroid Theory, Oxford University Press, New York, 1992.
[13] G.-C. Rota, Ten lessons I wish I had been taught, Notices Amer. Math. Soc. 44 (1997) 22–25.
[14] P. Seymour, Decomposition of regular matroids, J. Combin. Theory Ser. B 28 (1980) 305–359.
[15] A. Sokal, The multivariate Tutte polynomial (alias Potts model) for graphs and matroids, in: Surveys in Combinatorics 2005, in: London Math. Soc. Lecture Note Ser., vol. 327, Cambridge Univ. Press, Cambridge, 2005, pp. 173–226.
[16] R. Stanley, Modular elements of geometric lattices, Algebra Universalis 1 (1971/72) 214–217.
[17] W. Tutte, A ring in graph theory, Proc. Cambridge Philos. Soc. 43 (1947) 26–40.
[18] H.S. Wilf, Which polynomials are chromatic? Colloquio Internazionale sulle Teorie Combinatorie (Roma, 1973), Tomo I, Atti dei Convegni Lincei, No. 17, Accad. Naz. Lincei, Rome, 1976, pp. 247–256.