

```
f0=: *:
f1=: +&_0.1j0.8@*:
f2=: +&0.1j0.6@*:
f3=: +&0.1j0.61@*:
f4=: +&0.1j0.62@*:
```

You should see that the results can depend on the constants in a very delicate manner. In Section 6.3 we investigate the Mandelbrot set, which is a kind of roadmap to the Julia sets. In particular, in that section we will see an example of the fact that the Julia sets share features with related regions of the Mandelbrot set.

6.2 Experiment: Julia sets for Elliptic Curves

Curves of the form $y^2 = a_0 + a_1x + a_2x^2 + x^3$ are known as elliptic curves. We will explore the escape time of iterating $f(x) = \sqrt{a_0 + a_1x + a_2x^2 + x^3}$ in the complex plane. We assume *complex_dynamics.ijs* has been loaded and that `pal2` from Section 6.1 is available. We begin with an illustration and then proceed to consider how to experiment in order to find choices for the parameters that give interesting images.

```
f=:[: %: 0 0.4j0.1 0 1&p.
b=:f escapetc (10 255) 500 zl_clur _1.5 1.5j1.5
view_image pal2;b
```

The result is shown in Figure 6.2.1. We next turn to trying to find interesting examples by randomly choosing the coefficients and seeing whether the result is interesting. We trim our search by creating small images and then demanding that some number of pixels are white and that there is a decent variety of escape times. The function `mk_gis` displayed below does that. It was loaded by *complex_dynamics.ijs*. Its right argument has two bounds, the minimal number of distinct escape times and the minimal number of white pixels. If the small image created passes those tests, then a larger escape time array is computed. The left argument gives a scaling factor that can be used to bias the search. The result is the number of distinct escape times in the larger image and a display of the image. The global results include `coef` which gives the coefficients of the radicand, the function `f` that was iterated, and `b`, the array of indices.

```
load 'numeric'
randomize ''
```



Figure 6.2.1 Escape Time for an Elliptic Curve

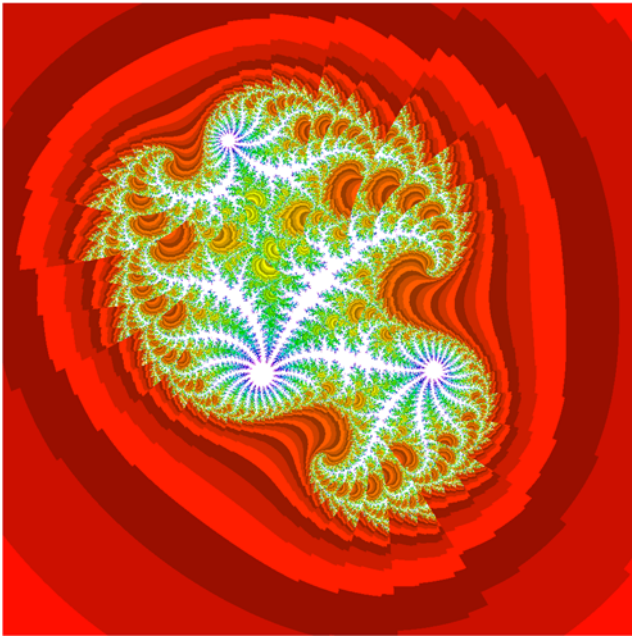


Figure 6.2.2 Escape Time for a Random Elliptic Curve

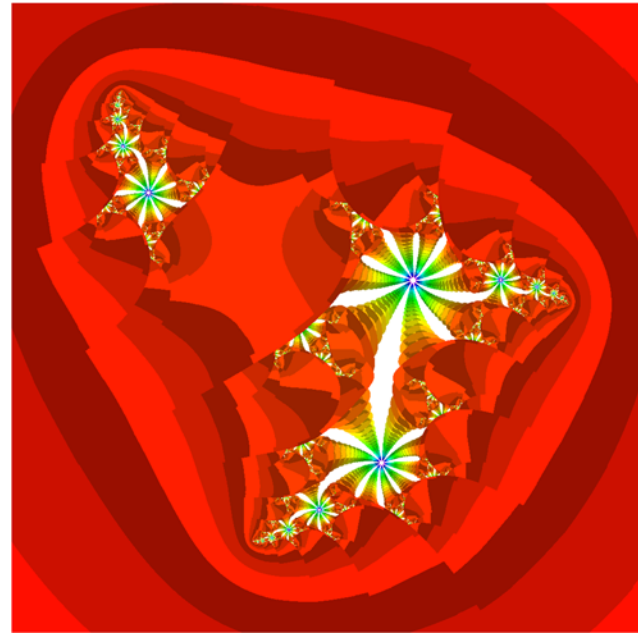


Figure 6.2.3 Escape Time for Another Random Elliptic Curve

```

mk_gjs                                     view the definition
3 : 0
1 mk_gjs y
:
'nmb rqw'=.y
whilst. (nmb>#~.,b) +. rqw>+/255=,b do.
  coef=: 1,~x*j./-/?2 2 3$0
  f=:[: %: coef&p.
  b=:f escapetc (10 255) 100 z1_clur _1.5 1.5j1.5
end.
b=: f escapetc (10 255) 500 z1_clur _1.5 1.5j1.5
view_image pal2;b
#~.,b
)

0.8 mk_gjs 100 10
view_image pal2;b

```

We repeated computations a couple dozen times, and 3 of the more interesting results appears in Figure 6.2.2, 6.2.3, and 6.2.4. There are also other ways to find interesting parameters as we will see in the next section.

6.3 The Mandelbrot Set

The Mandelbrot set is an escape time image of a different type. The escape time associated with a position c in the complex plane is the escape time of 0 under the iteration of the function $c + z^2$. Unlike Julia sets, position in the plane corresponds to different functions. There is a sense in which the Mandelbrot set gives a kind of road map for Julia sets.

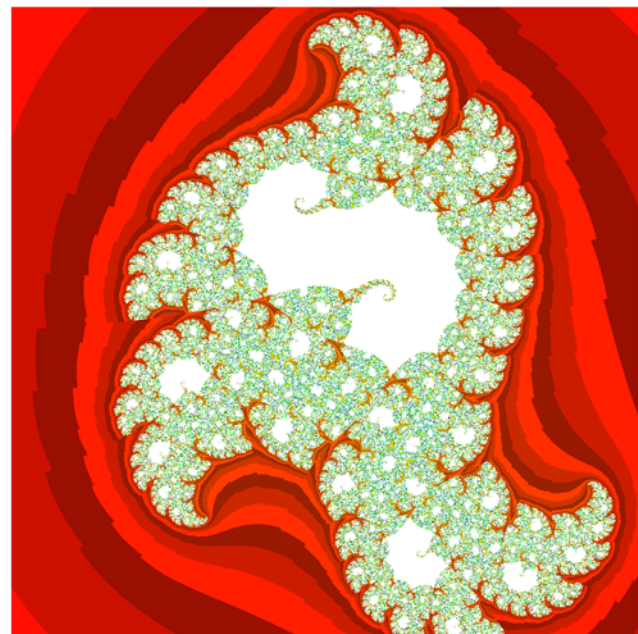


Figure 6.2.4 Escape Time of Another Random Elliptic Curve